Modeling the Pulse Line Ion Accelerator (PLIA): an algorithm for quasi-static field solution

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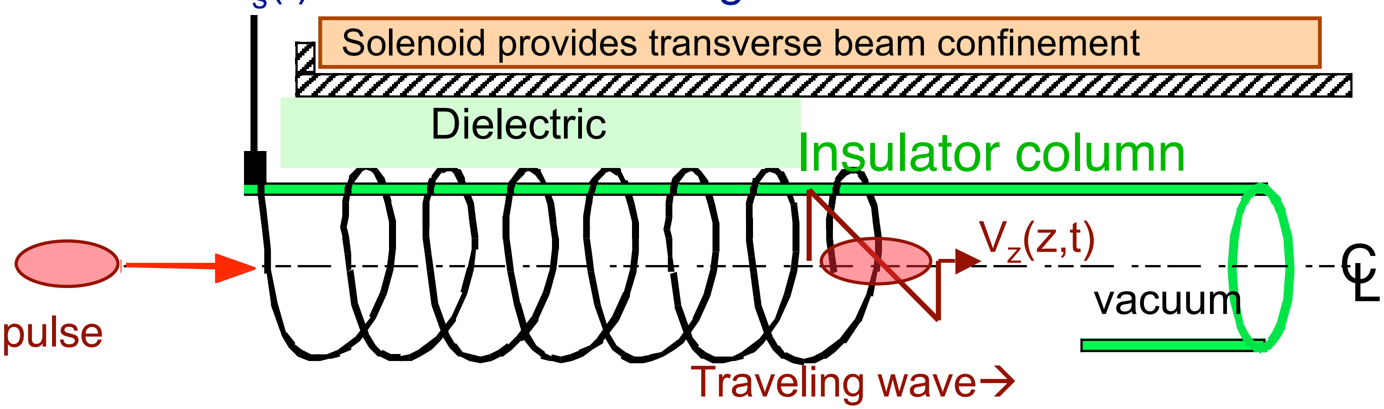
Abstract

The Pulse-Line Ion Accelerator (PLIA) is a helical distributed transmission line. A rising pulse applied to the upstream end appears as a moving spatial voltage ramp, on which an ion pulse can be accelerated. This is a promising approach to acceleration and longitudinal compression of an ion beam at high line charge density. In most of the studies carried out to date, using both a simple code for longitudinal beam dynamics and the Warp PIC code, a circuit model for the wave behavior was employed; in Warp, the helix I and V are source terms in elliptic equations for $\boldsymbol{E}$ and $\boldsymbol{B}$. However, it appears possible to obtain improved fidelity using a “sheath helix” model in the quasi-static limit. Here we describe an algorithmic approach that may be used to effect such a solution.

The Pulse Line Ion Accelerator (PLIA) is a new invention* which may prove attractive for High Energy Density Physics and Inertial Fusion Energy

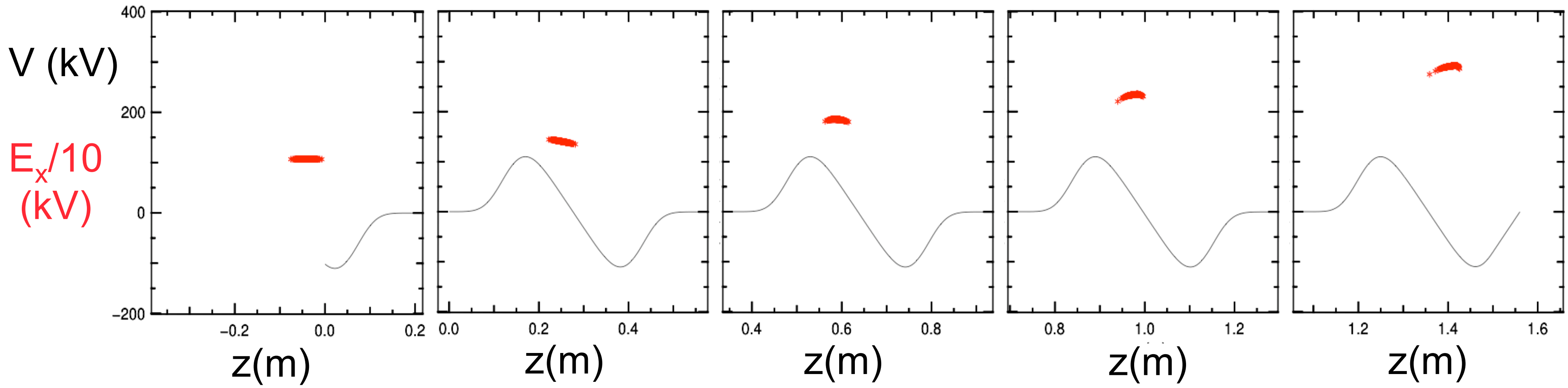


- Helix is a slow-wave structure
- Beam "surfs" on traveling pulse of E_z (circuit speeds of 0.01c to 0.3c OK)
- Energy increases >> input voltage to the helix are possible
- Potential for high current (limited by beam loading and gas-electron effects)

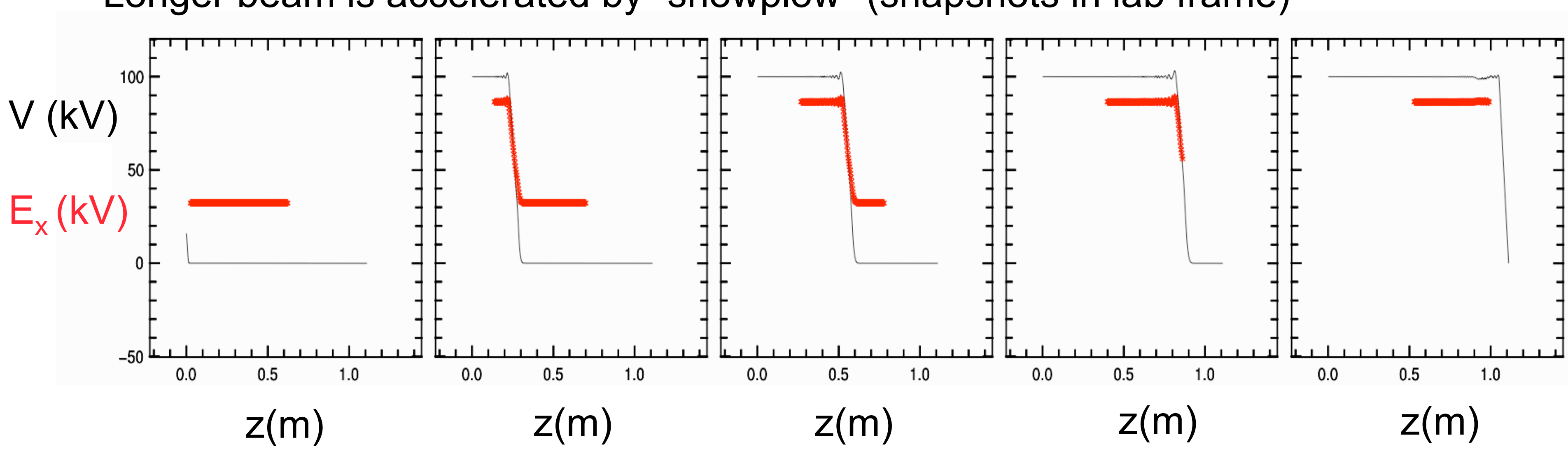
* R.J. Briggs, et al., LBNL patent disclosures 2004

PLIA can be operated in a short-pulse “surfing” mode or a longer-pulse “snowplow” mode

Short beam “surfs” on traveling voltage pulse (snapshots in wave frame)

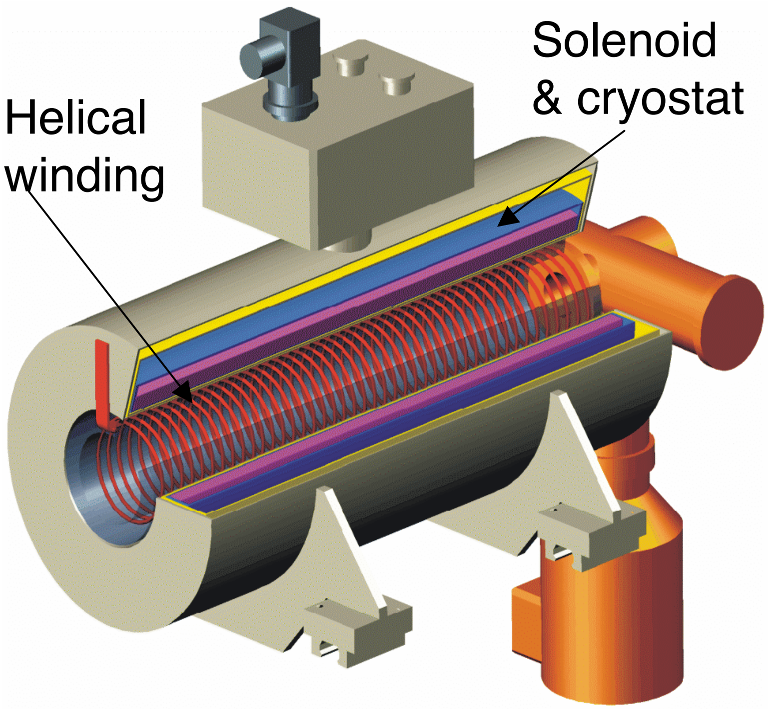


Longer beam is accelerated by “snowplow” (snapshots in lab frame)

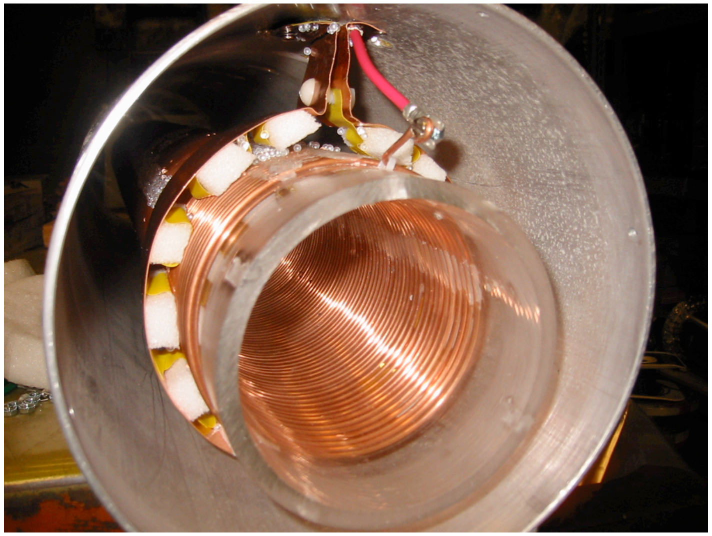


A PLIA may serve as front-end to an upgraded NDCX; such a system may offer advantages over pure induction

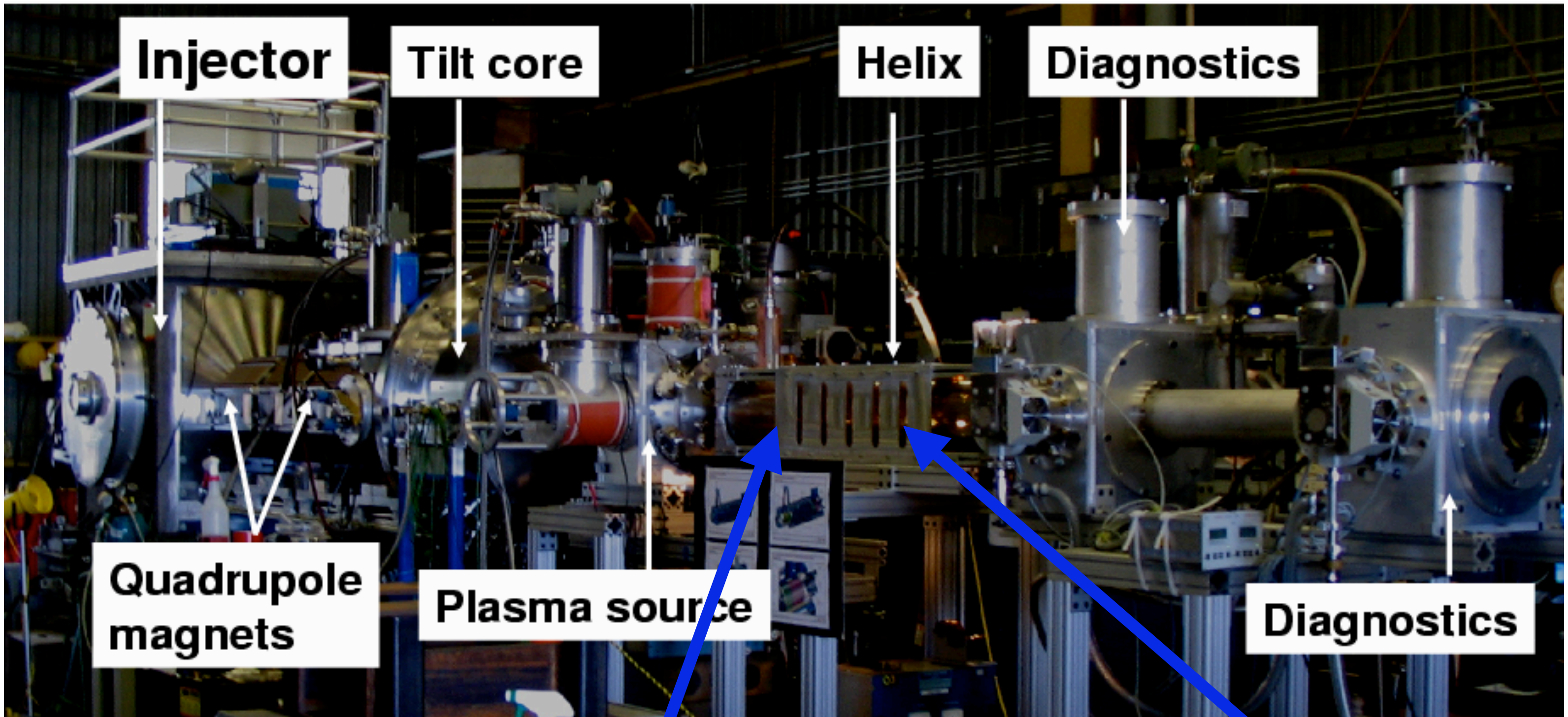
Accelerator cell concept



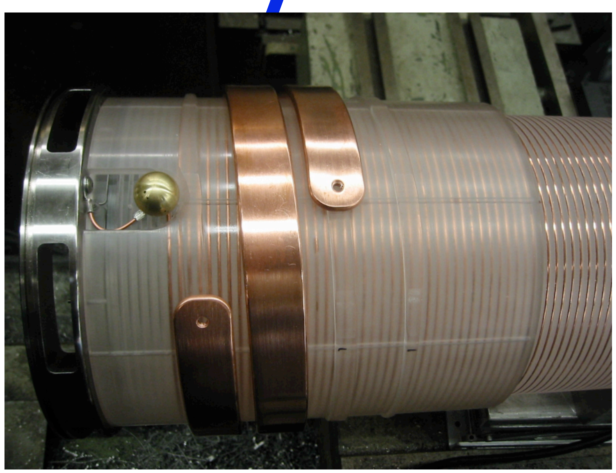
Compact transformer coupling (5:1 step-up)

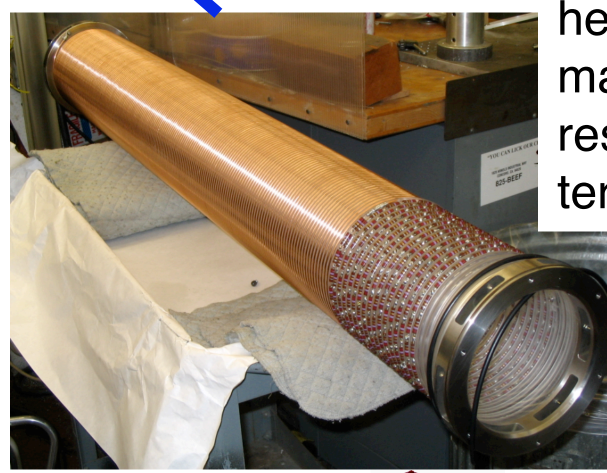


Pulse Line Ion Accelerator Experiment (PLIA) on NDCX



transformer coupling with 2-turn primary (step-up)

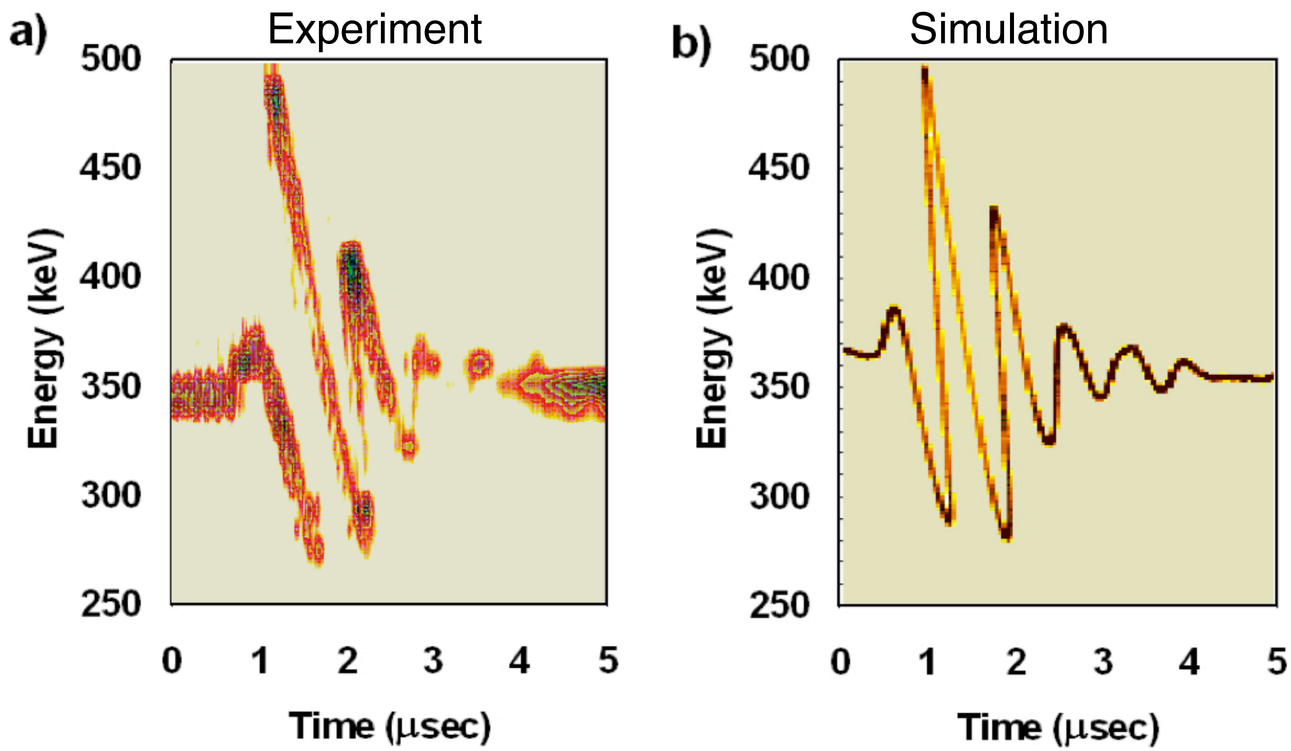




helix with matched resistive termination

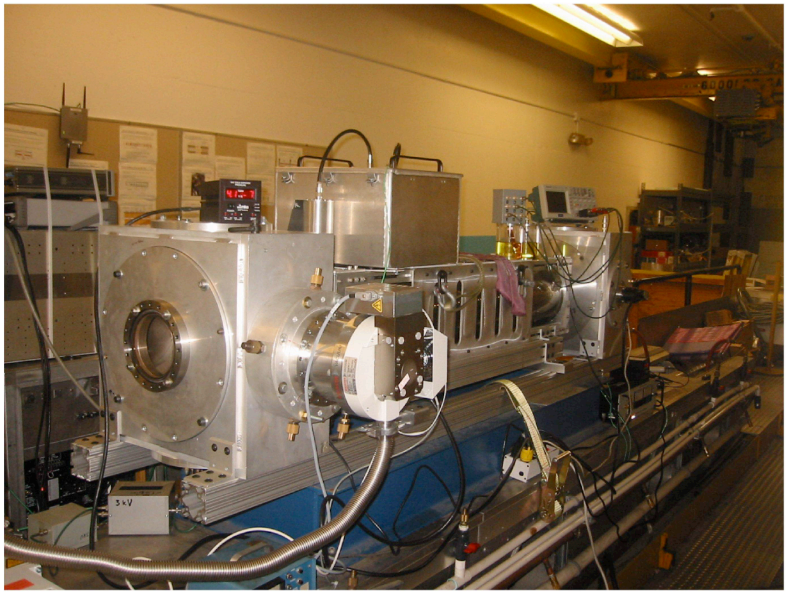
The beam energy was modulated by a ringing waveform & measured by an energy analyzer; Warp runs agree (roughly)

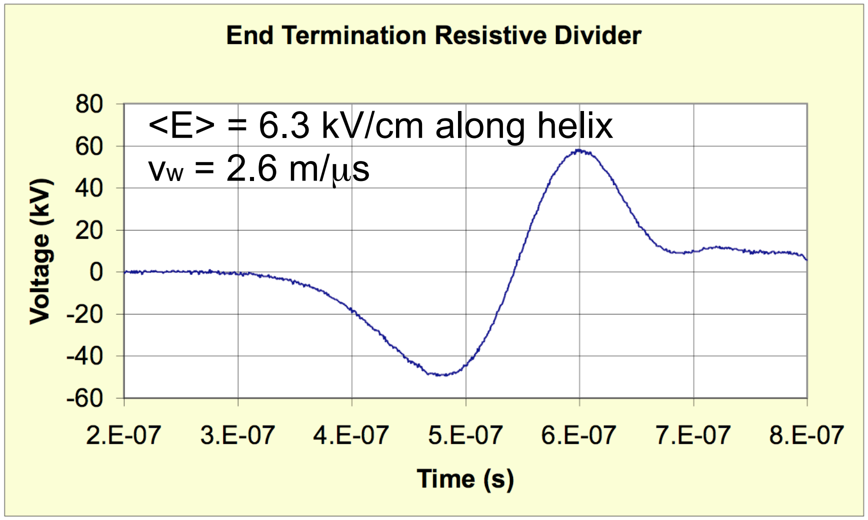
PLIA input voltage ranging from -21 kV to +12 kV
$\Rightarrow$ beam energy modulation ranging from -80 keV to +150 keV



- WARP run was based on V_{helix} measured at exit, then advected backward
- The oil dielectric / glass insulator helix used on these first tests had surface flashover problems at a few kV/cm. Flashover is avoided by use of a revised pulser which does not excite high frequency ringing

PLIA w/ oil dielectric & pyrex insulator has shown major gradient improvement on bench tests

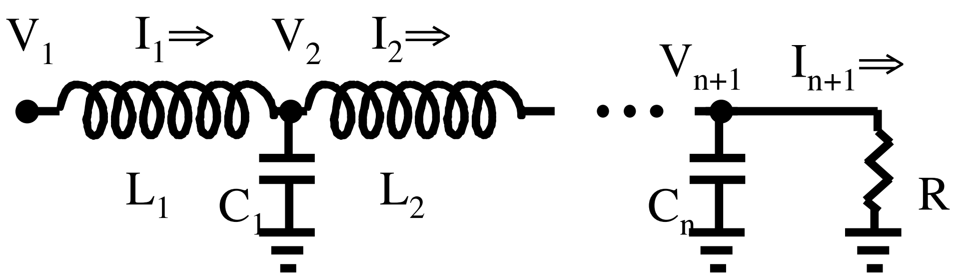




- Rings around OD of insulator (to suppress pulsed B inside, help grade voltage)
- New pulser circuit: filter to eliminate high level ringing @ ~30 MHz (20x fundamental frequency); damping prevents primary current reversal --> occasional discharges
- Gradient improved; here ~ 6 kV/cm

WARP includes a simple circuit model for the PLIA fields and the (self-consistent) beam fields

- Simplest model of PLIA treats it as a lumped-element transmission line:



$$C_i \frac{dV_{i+1}}{dt} = I_i - I_{i+1} + I_{b,i} - I_{b,i+1}$$

$$L_i \frac{dI_i}{dt} = V_i - V_{i+1}$$

- "Loading" of the helix voltage by the beam current is included in the circuit equations
- In WARP we solve via leapfrog, then get $\boldsymbol{E}$ and $\boldsymbol{B}$ by Poisson solves (using the V_i and I_i on the helix as sources)

Circuit speed $v_c = (\mathcal{L}\mathcal{C})^{-1/2}$

Impedance $Z = (\mathcal{L}/\mathcal{C})^{1/2}$

$\mathcal{L}$ = inductance / unit length

$\mathcal{C}$ = capacitance / unit length

- This has been a useful tool, but we seek a better model that can:
  - Account naturally for all mutual inductances and capacitances
  - Accurately capture "end effects," transformer coupling, wave dispersion

An improved circuit model* has been derived from "almost first principles" in the quasi-static limit

- Model is based on the "sheath helix" approximation
- Quasi-static approximation to the EM fields is valid because:
  - $v_c << c$
  - Helix radius a << free space EM wavelength
  - Dominant source of $\boldsymbol{E}$ is the *charge* on the helix wires (*not* the time derivative of the magnetic flux)
- Helix pitch $s / 2\pi a << 1$, where $s = 1/n$ is wire-center spacing
- This justifies a smooth approximation to the currents and charges on the helix, which are then used to calculate the fields
- Axial sheet current $K_z = I / 2\pi a$ << azimuthal sheet current $K_\theta = I / s$
- Also, $B_\theta << B_r, B_z$ by same ratio (pitch)
- Fields are axisymmetric; helix is a set of "hoops"
- Electrostatic potential yields $\boldsymbol{E}(r, z)$ and helix voltage $V(z, t) = \phi(r{=}a, z, t)$

* R. J. Briggs, "Electromagnetic fields of a helix, and related topics," LBNL Report LBID-2579 HIFAN 1513 (2005).

Here, we present an algorithm that may be used to solve for the fields (and the sources on the helix) in this approximation

- Charge continutity requires: $$\frac{\partial \lambda(z,t)}{\partial t} = -\frac{\partial I(z,t)}{\partial z} \qquad (1)$$
  where $\lambda(z,t)$ is the charge per unit axial length
- Define the surface charge density on the helix by $\sigma(z,t) = \lambda(z,t) / \; 2\pi a$; then, continuity becomes: $$\frac{\partial \sigma(z,t)}{\partial t} = -\frac{\partial K_z(z,t)}{\partial z} \qquad (2)$$
- We will use this to advance $\sigma(z,t)$ to the future time level, and so obtain the electrostatic potential and electric field in the simulation domain

The model includes a full set of mutual inductances and capacitances among the helix turns, and to ground

- To connect with the earlier circuit model, assume (for convenience) one computational helix node per turn: $\Delta z = s$. Integrate σ_i at each node i around the helix circumference, and over Δz, to get the node charge Q_i
- The Q_i are related to the voltages on the nodes by the mutual capacitances:

$$\sum_j C_{ij} V_j = Q_i \qquad (3)$$

- We may precompute the capacitance matrix C_{ij} , and invert it at each step to obtain the advanced-time voltages, after computing the source via the continuity equation
- In all versions of the circuit model, the capacitance between each helix turn and the grounded outer pipe (with dielectric in between) plays a major role. In the simplest model, the only nonzero elements of C_{ij} are these diagonal entries C_{ii} .
- Since $\sigma(z,t)$ only establishes the jumps in E_r and not the actual potential values, the outer-wall boundary condition in the Poisson solution used to establish C_{ij} serves to set the C_{ii}

The helix voltage $V(z, t)$ is related to the changing axial magnetic flux through the helix

- Taking a path of integration in Faraday's law that passes inside the helix wire, extending axially by a distance Δz, the flux $\Phi(z,t)$ is encircled $\Delta z/s$ times; the corresponding voltage change is:

$$\Delta V = -\frac{\Delta z}{s}\frac{\partial}{\partial t}\Phi(z,t) \qquad (4)$$

where the *total* flux through the helix (due to its own current and that of any driving "primary" winding) is:

$$\Phi(z,t) = \int_0^a B_z(r,z,t)\, 2\pi r\, dr \qquad (5)$$

- In the continuum limit,

$$\frac{\partial}{\partial t}\Phi(z,t) = -s\frac{\partial V(z,t)}{\partial z} \qquad (6)$$

Knowledge of the fluxes is enough to specify $B(r,z,t)$

- It is the magnetic field, not the flux, that is desired at the advanced time
- Initially it was not clear that the flux $\Phi(z,t)$, (a scalar function of z) contains enough information to uniquely specify $\mathbf{B}(r,z,t)$ (a vector function of r and z).
- A key realization was that a set of N fluxes can uniquely specify a set of N current sources (corresponding to N turns of the helix, when $\Delta z = s$), and then those sources can be used to compute $\mathbf{B}(r,z,t)$

- The azimuthal magnetic field component, $B_\theta(r,z)$, is neglected; as seen in direct solutions of the Maxwell equations [S. Nelson, Proc. PAC05], it is small near the beam.

  Thus the current sources are assumed purely azimuthal (circular hoops); at each computational node on the helix, the source is:

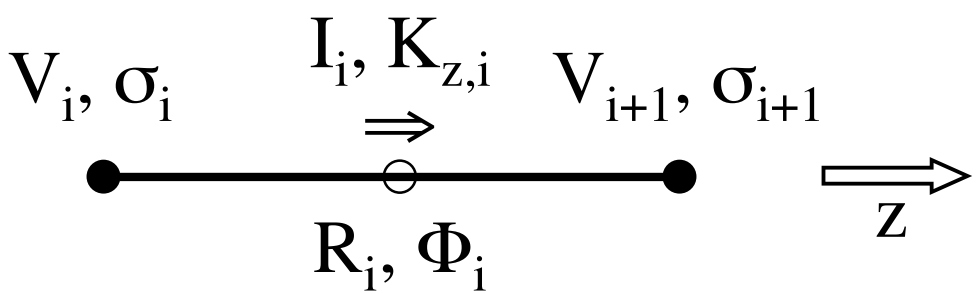
$$K_\theta(z,t)\Delta z = (2\pi a/s)K_z(z,t)\Delta z = I(z,t)\Delta z/s$$

An attractive finite-difference scheme interleaves the locations of the I's and the V's

- The voltage change from node i to node $i+1$ is related to the currents at the nodes by:

$$\Delta z \sum_j M_{ij} \frac{dK_\theta}{dt} = \frac{\Delta z}{s} \sum_j M_{ij} \frac{dI_j}{dt} = V_{i+1} - V_i \tag{7}$$

- In the circuit model, the current I_i flows between "voltage" nodes i and $i+1$.
- It is desirable to avoid two-cell difference expressions such as V_{i+1} - V_{i-1}. The Yee discretization of electromagnetics uses "staggered" grids for $\boldsymbol{E}$ and $\boldsymbol{B}$, so as to preserve important properties of the continuum equations.
- By analogy, we "offset" the axial locations of the currents from those of the voltages by half a cell. By examination of the governing equations, the "centering" of all other quantities can be developed:



These expressions relate to mutual-inductance formulas

- When $\Delta z = s$, Eq. (7) describes the usual mutual-inductance relationship. Most of our work with the circuit model used the simpler local self-inductance coupling, $L_i = - M_{ii}$
- For any node spacing, equations (6) and (7) are equivalent (in a finite-difference sense) when:

$$\sum_j M_{ij} I_j = -\Phi_i \tag{8}$$

which is a familiar formula for mutual inductances

- In practice, the M_{ii} are pre-computed by setting I_j to unity for each j in turn, with the other currents zero, computing $\boldsymbol{B}(r,z)$, and measuring the fluxes $\Phi(z)$ at all axial nodes

Code zoning need not be related to the helix wire spacing

- The model does not require $\Delta z = s$
- To allow arbitrary zoning in the simulation code, we interpret the mutual inductances as coupling the magnetic fluxes through the tori associated with the individual computational zones with the currents in those zones
- Note that we associate I_j with the current flowing in the helix wire through the plane $z=z_j$

  I_j is *not* the total current flowing azimuthally in computational zone j on the helix, and *is* insensitive to zone size when sufficiently small zones are employed

  To avoid confusion on this point, we rewrite Eq. (8) as:

$$2\pi a \sum_j M_{ij} K_{z,j} = -\Phi_i \qquad (9)$$

We proceed to describe an algorithm and some variations

The algorithm consists of:

- A pre-computation phase to establish the matrices that are needed during the time-advance: C_{ij} and M_{ij}
- A series of actions at each computational time step, to advance the system through an interval Δt
  - Denoting the time level (abbreviated "tl") of a quantity by a superscript, the step is described herein as an advance of the system from tl 0 to tl 1
  - The sheet current K_z is advanced from tl 1/2 to tl 3/2; it can be obtained at tl 1 by interpolation, for diagnostics or computation of $\boldsymbol{B}$
  - The overall procedure is formally a "leap-frog" advance, and is "time centered," reversible, and second-order accurate
- At start-up, the current in the helix may be assumed to be zero, or a half-step may be taken to obtain values of K_z at tl 1/2

The various possible levels of description call for different time-advancement algorithms

- If the goal is an improved circuit model, or a simplified 1-D discrete-particle or Vlasov model, inversion of the capacitance matrix suffices
  - An enhanced 1-D particle model might use applied fields computed via a separately precomputed Green's function that relates a unit voltage on the helix at $z=z_0$ to a voltage pattern $V(z-z_0)$ averaged over a nominal beam cross-section.
  - Beam self forces might be modeled using a simple "$-g d\lambda / dz$" formulation
  - Or (better), they might use yet another precomputed Green's function averaging across a nominal beam cross-section for both the charge-density and the force on a "slice" as a function of z
- To advance the particles in an (r,z) or 3-D simulation, it is necessary to obtain the electric field structure
  - Rather than using the capacitance matrix, a Poisson solution may be carried out at each step, including the beam charge as a source term
  - In this case, precomputation of C_{ij} may be unnecessary

Time advance

Enter the time step with $\sigma(z)$ defined at tl 0 and K_z defined at tl 1/2

(a) Advance σ using the continuity equation:

$$(\sigma_i^1 - \sigma_i^0)/\Delta t = -(K_{z,i}^{1/2} - K_{z,i-1}^{1/2})/\Delta z \tag{10}$$

(b, improved circuit model) Associate charges $Q_i = \sigma_i \Delta z$ with each node, and solve Eq. (3) for the V_j values

(b, full simulation case) The surface charge induces a jump in E_r at the helix:

$$\epsilon_{\text{out}} \left[\frac{\partial \phi^1(r,z)}{\partial r} \right]_{r=a^+} - \epsilon_{\text{in}} \left[\frac{\partial \phi^1(r,z)}{\partial r} \right]_{r=a^-} = -\sigma^1(z) \tag{11}$$

where ε is the dielectric constant, and “out” and “in” denote $r > a$ and $r < a$, respectively

There are multiple possible approaches to solving for the potential …

Time advance - potential in full simulation case - option (i)

(i) Solve coupled Laplace equations with this jump in $\partial\phi/\partial r$ as a constraint, and the additional constraint that $\phi(a)$ in the two subdomains be equal

Note: can't just solve separate Laplace equations in the inner and outer regions, with Neumann boundary conditions at the helix, since $\partial\phi/\partial r$ itself is not known in advance

Time advance - potential in full simulation case - option (ii)

(ii) "Smear" the surface charge by defining a charge density in the computational cell at the $r = a$ as $\rho = \sigma\Delta r$, then solve a Poisson equation (including source terms from helix charge, beam particles, stray electrons, and any other sources):

$$\nabla \cdot (\epsilon \nabla \phi^1) = -\rho^1 \qquad (12)$$

where the "stencil" for cells with radial index k corresponding to the helix radius might be:

$$\frac{\epsilon_{\text{out}}(\phi^1_{k+1} - \phi^1_k) - \epsilon_{\text{in}}(\phi^1_k - \phi^1_{k-1})}{\Delta r^2} + \left(\frac{\partial^2 \phi}{\partial z^2}\right) = -\frac{\sigma^1}{\Delta r} \qquad (13)$$

For an infinitesimally thin sheet, the $\partial^2\phi/\partial z^2$ term is not needed since all z variations are "slow" in comparison to jump in the radial electric field; but for a finite-thickness layer it is appropriate to include it.

Time advance - steps (c) - (e)

(c) From the solution $\phi(r,z)$ to the Poisson equation, the electric field $\boldsymbol{E}^1(r,z)$ can be obtained via finite differences, and the voltages $V^1(z)$ at the computational nodes on the helix obtained as the corresponding values of $\phi^1(r=a,z)$

(d) Time-advance the magnetic flux through each helix node i using a finite-difference form of Eq. (6):

$$(\Phi_i^{3/2} - \Phi_i^{1/2})/(\Delta t) = -s(V_{i+1}^1 - V_i^1)/\Delta z \qquad (14)$$

(e) Obtain currents at the helix nodes from the magnetic fluxes by inverting the inductance matrix, Eq. (9)

Time advance - combination of steps **(d)** and **(e)**

- **(de′)** Steps **(d)** and **(e)** may be combined, and introduction of the intermediary flux quantities Φ avoided, by substituting Eq. (9) into Eq. (14) to yield:

$$\frac{\Delta z}{s}\frac{2\pi a}{\Delta t}\sum_j M_{ij}(K_{z,j}^{3/2} - K_{z,j}^{1/2}) = (V_{i+1}^1 - V_i^1) \qquad (15)$$

  which is solved for $K_z^{3/2}$

  (M_{ij} is dense, with elements diminishing away from the diagonal)

- This approach is especially attractive when the helix is terminated in a helical resistive line, so that the voltage drop per unit length has both inductive and resistive contributions in series (since the current in the inductor equals that in the resistor). The voltage drop between nodes i+1 and i separated by a helix segment with a resistance per unit length R_i has two contributions:

$$\frac{\Delta z}{s}\frac{2\pi a}{\Delta t}\sum_j M_{ij}(K_{z,j}^{3/2} - K_{z,j}^{1/2}) + 2\pi a \Delta z R_i \frac{(K_{z,i}^{3/2} + K_{z,i}^{1/2})}{2} = (V_{i+1}^1 - V_i^1) \qquad (16)$$

Time advance - magnetic field, and "drive"

- **(f)** Use the resulting $K_{z,j}$ as sources to obtain $\boldsymbol{B}^{1}(r,z)$ or $\boldsymbol{B}^{3/2}(r,z)$ (when this field is needed, *e.g.*, when electron orbits are being computed, or for improved accuracy in ion orbits)

- The "drive" can enter in any of several ways:
  - (i) If it is a current source, it replaces step **(e)** at node 1 using the prescribed input $I_{\mathrm{input}}(\mathrm{t})$
  - (ii) If it is a voltage source, it enters as an internal boundary condition in the Poisson solution of step **(b)**
  - (iii) If it is via a transformer primary, itself driven by a current source, it becomes an extra "node 0" in the mutual inductance matrix

Discussion

- The equations presented herein define a detailed one-dimensional model of wave propagation on the helix that can be coupled with a one-dimensional particle-in-cell model.

  The simplicity of such a model is attractive for development of insight and for rapid scoping studies; indeed, the earlier (simpler) circuit model lent valuable insight into the behavior of this novel system.

- That earlier circuit model was adapted for use in Warp, which allows multi-dimensional (2-D and 3-D) particle-in-cell simulations to be carried out, including detailed space charge fields.

  Since an improved model is desired for design and analysis studies, we hope to implement this improved field model in Warp.

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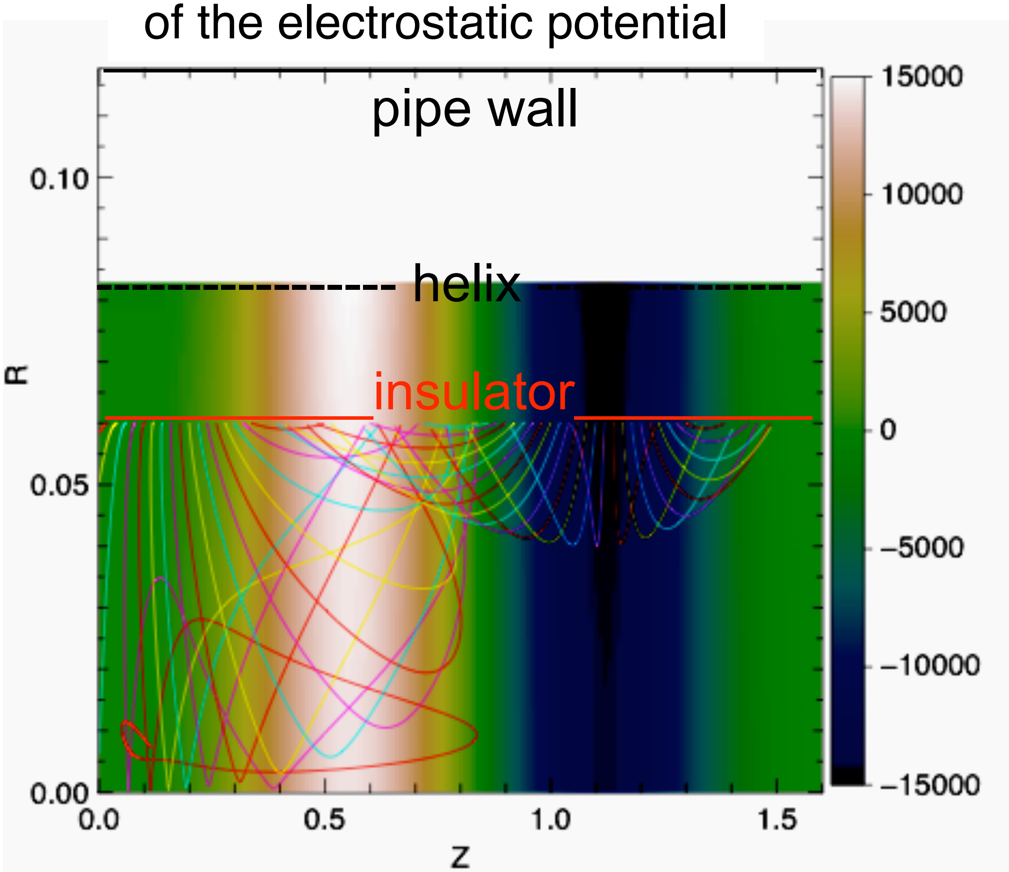
Extras

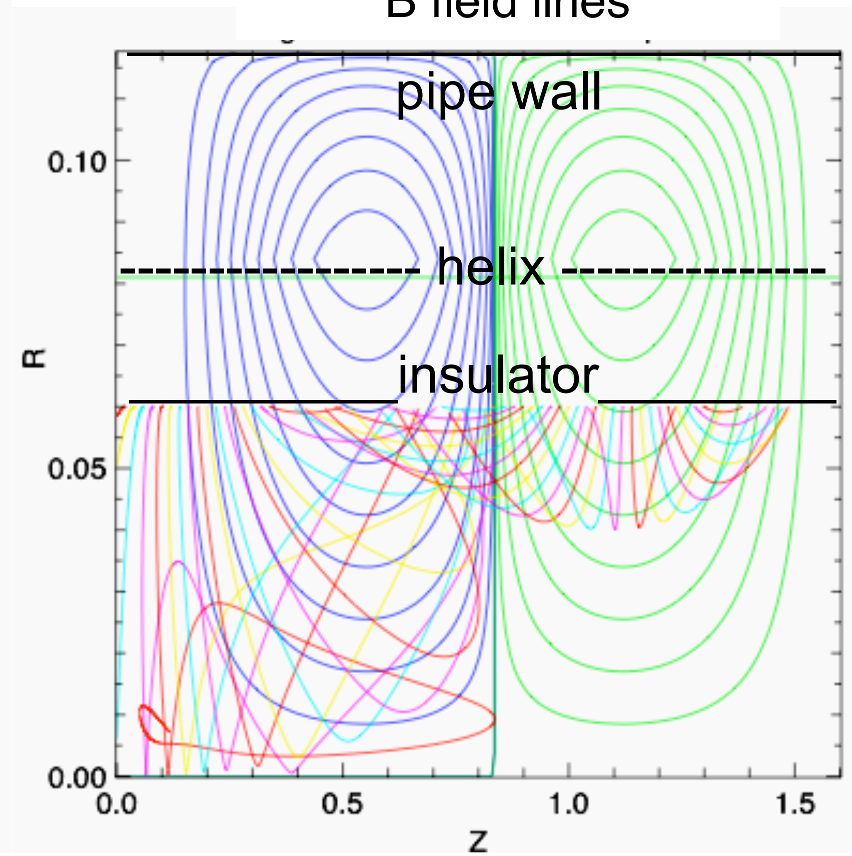
To understand the breakdown, we are using WARP to follow "tracer" electrons emitted from the insulator surface

Tracers overlaid on contours of the electrostatic potential

Time = 135 ns

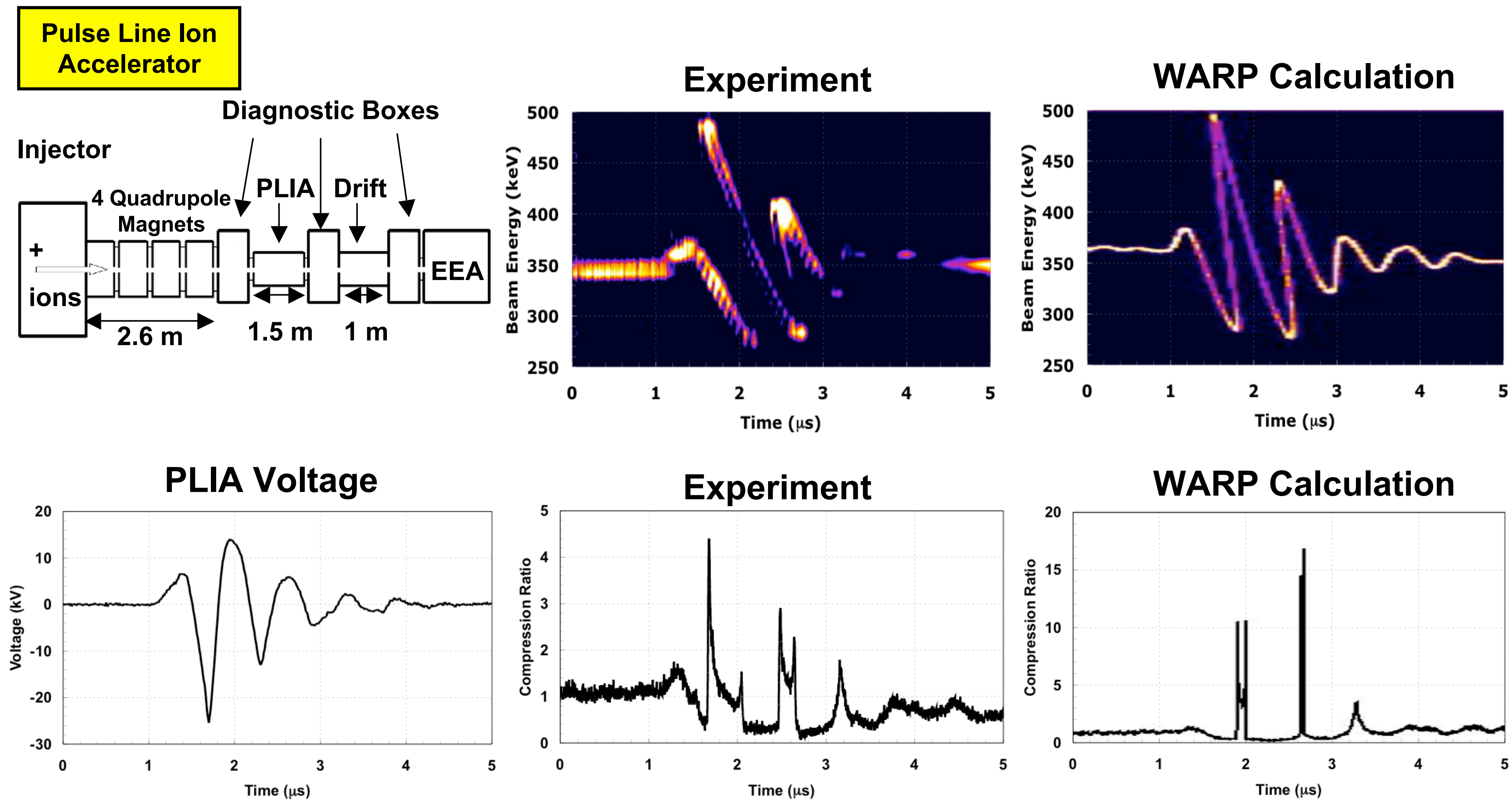
Tracers overlaid on B field lines





- Tracers are emitted with zero velocity in the lab frame
- Helix fields are "frozen" in the wave frame (assumes dispersionless wave)
- These images are in the wave frame

PLIA test displayed acceleration, deceleration, and longitudinal bunching



Slide courtesy Josh Coleman